UH-6165

B. E. II (Sem. III) (Comp. I.T.) Examination

May / June – 2012

Discrete Mathematics

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :	☛ B. E. II (Sem - III) (Comp. I.T.)
Name of the Subject :	☛ Discrete Mathematics
☛ Subject Code No. :	6 1 6 5
☛ Section No. (1, 2,.....) :	Nil

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures to the right indicate full marks.

1 (a) Do as directed : **10**

(i) Define isomorphic graphs.

(ii) Define path and circuit.

(iii) Define Euler graph.

(iv) Define radius and diameter of a tree.

(v) Define a cut set.

(b) Attempt any two : **10**

(i) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

(ii) Prove that the number of vertices of odd degree in a graph is always even.

(iii) Prove that a simple graph with n vertices and K components can have atmost $\frac{(n-k)(n-K+1)}{2}$ edges.

2 (a) Describe Konigse-berg bridge problem. **3**

(b) Attempt any three : **12**

(i) Prove that a connected graph G is a tree if and only if adding an edge between any two vertices in G creates exactly one circuit.

(ii) Prove that every tree has either one or two centres.

(iii) Prove that every connected graph has atleast one spanning tree.

(iv) Prove that a graph is a tree if and only if it is minimally connected.

3 (a) Describe three-utilities problem. **3**

(b) Attempt any three : **12**

(i) Prove that Kuratowski's 2^{nd} graph is non-plannar.

(ii) Define a plannar graph. Prove that a connected plannar graph with n vertices and e edges has e-n+2 regions.

(iii) Write properties common to the Kuratowski's two graphs.

(iv) Prove that a vertex V in a connected graph G is a cut-vertex if and only if there exist two vertices x and y in G, such that every path between x and y passes through V.

4 (a) Do as directed : **10**

(i) In usual notations, define $P \rightarrow Q$, write its negation also.

(ii) Define a group.

(iii) Draw Hasse diagram of (S_{45}, D), where S_{45} is the set of divisors of S_{45} and D stands for divisors.

(iv) Show that $f(x,y) = x^y$ is a primitive recursive function.

(v) What can you say about sets A and B if given that

(i) A-B = A

(ii) A U B = ϕ

(b) Attempt any two : **8**

(i) Prove that $(R, \leq)$ is a poset, where R is the set of real numbers and "$\leq$" is a binary relation on R.

(ii) Show that $(P \rightarrow Q) <=> (7P \ VQ)$

(iii) Let R denote a relation on the set of ordered pairs of positive integers such that (x,y) R (u,v) it xv=yu. Show that R is an equivalence relation.

5 (a) Attempt any two : **8**

(i) Prove that the order of a subgroup of a finite group divides the order of the group.

(ii) Let (S, *), (V, Δ) and (T, $\oplus$) be semi groups and $g : S \to V$ and $h: V \to T$ be semigroup homomorphisms. Then prove that (hog) : $S \to T$ is a semigroup homomorphism from (S,*) to (T, $\oplus$).

(iii) Prove that a subset $S \neq \phi$ of a group G is a subgroup of (G, *) if and only if for any pair of elements a, b $\in$ S, $a*b^{-1} \in S$.

(b) Attempt any two : **8**

(i) Draw Hasse diagram of (S_{60}, D), where S_{60} is the set of divisors of 60 and D stands for divides.

(ii) Let <L, $\leq$> be a lattice in which * and $\oplus$ denote the operation of meet and join respectively. Then for any a,b $\in$ L, show that $a*b = a \iff a \oplus b = b$.

(iii) Show that a lattice with 3 or fewer elements is a chain.

6 (a) Attempt any two : **8**

(i) Define a complemented lattice. Find the complements of every element of the lattice (S_{70}, D).

(ii) In any Boolean Algebra, show that

(a) $a + (a'b) = a+b$

(b) $(abc) + ab = ab$

(iii) Prove that every chain is a distributive lattice.

6 (a) Attempt any two : **8**

(i) Show that the function f(x) = [x/2] which is equal to the greatest integer which is $\leq$ x/2, is primitive recursive.

(ii) Show that f(x,y) = x+y is primitive recursive.

(iii) Use the Karnaugh map representation to find a minimal sum-of-produts expression for the function

$f(a,b,c,d) = \sum (0,2,6,7,8,9,13,15)$